

same shape all along, it satisfies Leibnitz's definition of a straight line, and it is, in fact, a geodesic line of the surface.

Hence we have this second proposition—that all points in the surface opposite to a given point lie in a straight line.

From the method of its construction, this straight line is farther from the given point than any other line in the surface. Travelling from the given point as a centre, in whatever direction we might set out, we should, after completing half our journey, arrive at this farthest straight line, we should cross it at right angles, and we should then keep getting nearer and nearer to our starting-point, until we finally reached it from the opposite side.

Each separate point in the surface, moreover, has a separate farthest line. For if any two points be taken, the points opposite to them on the straight line which joins them will be distinct. Hence their farthest lines will cut this joining line in two separate points. They must, therefore, be two *separate* lines, for the same straight line cannot cut another straight line in two separate points. In a similar manner it may be shown that each straight line in the surface has a separate farthest point. Hence there exists a reciprocal relation between the points and straight lines of the surface, a relation which we may express by saying that every point in the surface has a *polar*, and that every straight line in the surface has a *pole*. It is then easy to show that when a point is made to move along a straight line its polar will turn about a point, and that when a straight line is made to turn about a point, its pole will move along a straight line.

It is interesting to compare these propositions with the corresponding ones in spherical geometry. There, too, each point has a farthest geodesic line; that is to say, a geodesic line which is farther from it than any other geodesic line on the sphere. But each geodesic line has *two* farthest points or poles, instead of having only one. Hence there is not that perfect reciprocity of relationship between points and geodesic lines which exists in the surface we have been examining; and this is one of the many ways in which the sphere shows itself to be inferior to that surface in simplicity.

The most astounding fact I have elicited in connection with this surface is one which comes out in the theory of the circle. Defining a circle as the locus of points equidistant from a given point, we shall find that it assumes a very extraordinary shape when its radius is at all nearly equal to half the entire length of a straight line. For let us again figure to ourselves a number of straight lines radiating from a point. Let l be the total length of each straight line. Then the supposition we have to make is that the radius of our circle shall be nearly equal to $\frac{l}{2}$. Let us suppose it equal to $\frac{l}{2} - m$, where m is small as compared with l . Each of the radiating lines will cut the circle in two points, and each of these points will be at a distance from O equal to $\frac{l}{2} - m$ or $\frac{l}{2} + m$,

according as the distance is measured in the one direction or the other. And their distance from each other will be equal to $2m$, that is to say, it will be comparatively small. But each point on the polar of O will be at a distance from O equal to $\frac{l}{2}$. Hence each point on the circle will

be at a distance from this polar equal to m . Moreover, every point at a distance of m from the polar will be a point on the circle, because it will be at a distance of $\frac{l}{2} - m$

from O . But the locus of points at a distance of m from the straight line, AB , will consist of two branches, CD and EF , one on either side of AB , and at the same distance from it along their whole length. It is true that these branches form in reality a single continuous line.

A point travelling along from C to D , and further in the same direction, would ultimately appear at E , travel along to F , and then, after a further journey, reappear at the point C . But this does not alter the fact that when a small portion only of this line is contemplated, it presents the appearance of two straight lines, each of them parallel to, and equidistant from, AB .

In the limiting case, where the radius becomes equal to $\frac{l}{2}$, CD and EF both of them coincide with AB . The

circle merges into a straight line, and becomes, in fact, the polar of its own centre. It is not, indeed, quite accurate to say that it merges into a straight line, for it reduces itself rather to two coincident straight lines, and its equation in co-ordinate geometry would be one of the second degree.

In regard to the surface here treated of, it is easy to see that, as with the sphere, the smaller the portion of it we bring under our consideration, the more nearly its properties approach to those of the plane. Indeed, if we consider an area that is very small as compared with the total area of the surface, its properties will not differ sensibly from those of the plane. And on this ground it has been argued that the universe may in reality be of finite extent, and that each of its geodesic lines may return into itself, provided only that its total magnitude be very great as compared with any magnitude which we can bring under our observation.

In conclusion, I cannot do better than quote the passage in which Prof. Clifford explains what must be the constitution of space if this hypothesis should be true. "In this case," he says, "the universe, as known, is again a valid conception, for the extent of space is a finite number of cubic miles. And this comes about in a curious way. If you were to start in any direction whatever and move in that direction in a perfect straight line according to the definition of Leibnitz, after travelling a most prodigious distance, to which the parallax unit—200,000 times the diameter of the earth's orbit—would be only a few steps you would arrive at—this place. Only, if you had started upwards, you would appear from below. Now one of two things would be true. Either when you had got half way on your journey you came to a place that is opposite to this, and which you must have gone through, whatever direction you started in, or else all paths you could have taken diverge entirely from each other till they meet again at this place. In the former case every two straight lines in a plane meet in two points, in the latter they meet only in one. Upon this supposition of a positive curvature the whole of geometry is far more complete and interesting; the principle of duality, instead of half breaking down over metric relations, applies to all propositions without exception. In fact I do not mind confessing that I personally have often found relief from the dreary infinities of homaloidal space in the consoling hope that, after all, this other may be the true state of things."

F. W. FRANKLAND

HYDROGRAPHY OF WEST CENTRAL AFRICA

MR. STANLEY'S second letter in last Thursday's *Telegraph* contains important information on the district between Tanganyika and the Albert and Victoria Nyanza—information complementary to that given in his former letters, which we embodied in a map, vol. xiv. p. 374. He has, in fact, discovered another "source" of the Nile, and one evidently of great length and volume—the Kagera—which he has gallantly named the Alexandra Nile. This river issues from a large lake, Akanyaru or Alexandra Nyanza, in two branches and flows north, uniting under 1° S. lat., and flowing east to the Victoria Nyanza. Mr. Stanley was only able to see the Alexandra Nyanza from a distance, but it is evidently of consider-

able size, and receives a river at its west end, the Upper Alexandra Nile, which probably comes from a considerable distance. Mr. Stanley believes that the Alexandra Nyanza has a marshy connection with Kivu Lake on the south, from which issues the Rusizi, an affluent of the Tanganyika. If then these various connections are ultimately verified, the problem of African hydrography becomes more complicated than ever. The Rusizi will connect the Nile system with Tanganyika, and very shortly, at least, Mr. Stanley believes, the Lukuga will carry the water of the latter to the west—to the Congo, say some. Meantime Mr. Stanley is probably at or has already left Nyangwe. After deciding this question of the connection of Albert and Tanganyika Lakes from that side, he will probably devote himself to the task of tracing down the Lualaba, which, according to Cameron, should bring him into early communication with Dr. Nachtigal, who is to trace up the Congo.

It may not be uninteresting to point out what is the present state of the problem which these two explorers have set themselves to solve. Our principal scientific authorities on the Congo are still Capt. Tuckey and Prof. Smith, who in 1816 ascended about 200 miles up the river, and who have left us a record yet deserving of study. They left England at a time when the outlet of Mungo Park's Niger was a subject of speculation, and amongst the theories then started, the Congo, as an outlet, held a high place. The same notions of the magnitude of this river obtained then, and Capt. Tuckey and his civilian scientific staff started with the idea that they would be able to navigate it for hundreds of miles. They had, however, only been in the river some four or five days when Prof. Smith makes this entry in his diary:—"The channel is very narrow and the current never more than three knots . . . everything yet seems to indicate that the descriptions of the great breadth of the river, the length of its course, &c., have been exaggerated." Again, twelve days afterwards, when they had got considerably further up the river, he writes, "The whole appearance of the river, its numerous sandbanks, low shore, inconsiderable current, narrow channel, seem but little to justify its extravagant fame. Its sources cannot be further inland than those of the Senegal and Gambia." Capt. Tuckey, who ties himself very rigidly to a statement of facts, ventures to say that at Fathomless Point the true mouth of the river "is not three miles in breadth; and allowing the mean depth to be forty fathoms and the mean velocity of the stream four and-a-half miles an hour, it will be evident that the calculated volume of water carried to the sea has been greatly exaggerated." The mean velocity of the current higher up the river than the true mouth appears to be about two miles, and Tuckey remarks that they found no difficulty in rowing the gigs to the foot of Casan Yellala *against the current*.

These falls or rapids (Yellala) deserve some notice. They extend continuously for about twenty miles along the river, and are very much like the rapids on the Somerset Nile between Foweira and Magungo, where Col. Gordon reports a fall of 700 feet in a space of ten or fifteen miles. On August 14, 1816, Prof. Smith says, "We discovered the celebrated fall of Yellala, at a distance of about a mile and a half. But how much were we disappointed in our expectations on seeing a pond of water only with a small fall of a few hundred yards." They had been led to expect a second Niagara, and instead of that, found a rapid having a perpendicular fall of thirty feet in a slope of 300 yards formed of a descending bed of mica slate. The width of the river is very various, sometimes expanding to half a mile. It is compared by Tuckey to Loch Tay and by Smith to the Drammen, in Norway, at the bridge. Sometimes it contracts to 100 yards; in one place it is reduced to fifty yards in breadth, but at this point the stream rushes through at the rate

of eight miles an hour. The rapid and considerable rise of the water during the rainy season is largely accounted for by the fact that "the hills do not absorb any of the water that falls, the whole of which is carried direct to the river by gullies and ravines, with which the hills are furrowed all over." These hills are composed entirely of slate, with masses of quartz and syenite, and their extreme barrenness forms one of the most striking features of the country.

It would appear from Capt. Tuckey's and Prof. Smith's reports that the farthest point they reached on the river was at least 1,000 feet above the sea, and as this point is about 800 miles in a direct line from Nyangwe, which Cameron has fixed at 1,400 feet, the connection between the Congo and Lualaba on the question of level alone seems very doubtful.

The Casai and Kwango are doubtless the chief affluents of the Congo; it may have tributaries from the north and north-west behind the coast ranges, but these will be of secondary importance. As soon as we get east of the Congo water-parting, we begin to descend to the great valley of the Lualaba, Livingstone's "central line of drainage." This river occupies the centre of a saucer-like depression, one lip being probably the Congo water-parting, the other the Bambarre, or perhaps Kabogo Mountains to the west of Tanganyika. The fall of this depression is from south to north; commencing at the Katanga copper mines of the Pombeiros, it runs to Lake Kassali 1,750 feet, to Nyangwe 1,400 feet; thence to the "Unvisited Lake" of Livingstone, the "Great Lake" of Poncet, or the "Sankorra" of Cameron, probably also the "Liba" of the Benin slaves, and so on by the Shari to Lake Chad, 830 feet.

From these statements, then, it will be seen that the solution of the hydrographical problem of Western Central Africa is difficult to arrive at on the data we at present possess, and that to advocate any special theory may be rash. The Congo theory is a fascinating one, but the levels seem against it. However, with two such men as Nachtigal and Stanley in the field, the solution of this problem, as of others almost equally interesting, will soon be discovered.

THE LONDON INDUSTRIAL UNIVERSITY

WE give below a series of extracts from an admirable letter addressed by Major Donnelly, the chief of the scientific staff of the Science and Art Department, to Sir Sydney Waterlow, with reference to the proposed Industrial University to be established by the City Guilds in London:—

London, March 14, 1877

DEAR SIR SYDNEY WATERLOW,—In reply to your request, I am happy to place at your service such suggestions, with regard to the proposed "City Guilds' Industrial University," as my experience in connection with the Science and Art Department enables me to offer. . . .

Under anything like a broad view of the subject it would be difficult to say what branch of learning should be omitted in an Industrial University. But if we confine ourselves to what is practicable with the probable means immediately at command, and if we look to commence by supplying that of which there is the greatest want, we shall, I think, have no hesitation—considering the relative facilities for obtaining instruction in the different branches of knowledge—in deciding that science as now understood, and particularly Applied Science, has the first call on our attention.

. . . The Industrial University might be commenced by establishing professorships with the necessary laboratories, tutorial staff, &c., in the following branches of Science and Art:—

- Mathematics (Pure and Applied and Practical Geometry).
- Chemistry.
- Physics (Heat, Light, Magnetism, and Electricity).
- Mechanics (Practical Mechanics, Machinery, and Machine Drawing).
- Engineering and Building Construction; and in